Conceptualising more explicit University-level oral Language Communication Training: CLIL Tutors as Mediators in the Language Classroom

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Abstract

Despite having reached the mandatory German language level requisite for entrance to university, many first-semester international students do not begin their studies with sufficient content language skills in their field of study. Prior studies have focused on the importance of language training with written texts and measured student performance based on these; however, few have investigated the importance of oral communication in the college-level CLIL environment. Focusing on the language of mathematics, this paper discusses how students’ oral communication needs are identified and then implemented in the content-specific language classroom. CLIL tutors, working alongside German language instructors, can provide learners with meaningful and authentic input related to effective oral communication skills in an academic setting. The results of student evaluations as well as open interviews with instructors and student tutors at a German technical university give insight into how German oral communication skills are being practiced in language for specific purposes (mathematics) classes.

Keywords: pre-service teacher, oral communication, LSP mathematics, ideolects, performative verbs, metadiscourse

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“No one hears as what he knows, no one hears as what he can feel, imagine and think” (Goethe) (Schmidt, 2015, p. 220).

“...all spoken language (in the classroom and elsewhere) provides only clues to its full meaning” (Lemke, 1990, p. 92).

Oral communication is a dubious undertaking because what we hear is not always the message that is being communicated. How many of us have caught ourselves singing along to a favorite tune on the radio, thinking we knew the lyrics and then much to our chagrin or amusement realizing that our rendition was quite different from the original:

This includes, for instance, a native German speaker hearing and then singing “Eichhörchchen-Sheriff” (squirrel sheriff) instead of Bob Marley’s “I shot the sheriff” (Hacke, 2004, p. 51) or another person chanting “She’s got a chicken to ride.” to the Beatles’ original lyrics “She’s got a ticket to ride” (Hacke, 2004, p. 50). While the reasons behind the disparate versions of what we hear and what we say are far too complex for the purposes of this paper, it is certainly worthwhile to recognize the fact that oral communication can present speakers with significant hurdles if what is being communicated to them is not being understood. A creative interpretation of a song may just remain a humorous situation; however, in an academic or professional context a misinterpretation such as this can have serious consequences for both the listener and the speaker.

The importance and value of oral communication cannot be underestimated in today’s university-level classroom. As language instructors, we witness the majority of our students rating practice of speaking skills in class as very high. This is not only so because our students are social beings and value communicating with one another. In the academic and
professional world, the importance of oral communication is greater than ever (Joughin, 2010, p. 1). In an increasingly global working world, effective oral communication skills are key to later career success. The communicative training grounds for young professionals are located in large part in schools and at universities. These institutions have the responsibility and opportunity to guide learners to become independent, articulate critical thinkers and speakers (Joughin, 2010, p. 1).

This paper first explores the tradition of oral communication in European education. It then moves on to a more detailed discussion of theories behind oral communication in a CLIL setting and aims to answer the question: If oral communication is so important in the professional world and is so embedded in European academic tradition, why is the written text still deemed by many to be “the” instrument for measuring the depth of student knowledge? The discussion explores the issue of implicit training in oral communication and pleads for more explicit treatment of content-specific oral communication training in the future. We report on the development of a content-specific language course for mathematics at a technical university in Germany. Content-specific language courses led by a team of language instructors and content-expert tutors can significantly increase the authenticity and meaningfulness of CLIL instruction for students. We share results of student evaluation reports, as well as reflections from teachers and tutors.

1 Oral communication and assessment in higher education in Europe

European education can be described by its “multilayered structure” (Segal & Vasilache, 2012, p. 514) and attributes influences from Greek, Roman as well as the influence of educational factors from the Orient/Far East, in particular Chinese, Jewish and Islamic traditions. The notion of apprenticeship, in which students learn from a master and engage in a “teaching-learning” dialogue (p. 515), is in large part based upon the Socratic Method. According to Segal, this is one of the main characteristics of the Greek and Roman educational tradition that has shaped
modern European education systems (p. 515). Cordasco describes the Socratic Method as “a process of obtaining a concept or definition inductively by conversation on moral and philosophic problems” (1963, p. 7). A further important contribution of Socrates to education includes the idea that “…knowledge is obtained objectively by conversation, and subjectively by the reflection and classification of one’s experiences” (p. 7).

Aristotle’s approach to education saw “Oral communication as a way of organizing knowledge” (Donskikh 2019, p. 5). The peripatetic approach to teaching required students to recall what was being said while in motion (walking). Learning and understanding required students to systematically remember what had been said. For most of us this type of learning is practically unimaginable in a society where “it is possible to find the right link, the right text in the book, or, even more effectively, at any time to turn to the help of a smartphone” (p. 7).

The oral transmission of knowledge also has its place in Islamic education traditions as students attended madrasas (kuttab) and surrounded their master as he relayed his knowledge orally (Segal & Vasilache, 2012, pp. 516-517). Education systems in the Middle East encouraged student-centered learning in a way that was “confluent, not competitive” – where learning took place in the form of dialogue, between teacher and student or amongst students (p 517).

The legacy of these classical education systems is clearly apparent in assessment of student progress in the European higher educational system. Oral exams are institutionalized in most secondary educational settings and are often a prerequisite for advancement to a tertiary level of academic qualification. For instance, in Germany, in order to be admitted to university, all students must undergo written and oral exams as part of the high school exit exam (Abitur). International students whose first language is not German must achieve a certain language level in order to gain admission (level B2/C1-European Framework of Reference for Languages) to university. Oral proficiency is tested explicitly. This is not the case in other countries such as the United States,
where there is no unilateral oral proficiency requirement for students to be admitted to university from high school.

Most university academic programs in Germany have institutionalized oral examinations. This was the case in the former three-tiered system in Germany, which was comprised of state examinations, Diplom and Magister, and has remained so even after the Bologna reform in 1999. Oral exams also serve as a part of the Bachelor’s or Master’s thesis, not to mention defense of the doctoral and post-doctoral theses (Habilitation) (Kehm, 2001, p. 25).

2 What conviction lies behind the oral tradition in European education?

The oral tradition is so ingrained in European education that it may at first be difficult to flesh out the reasons behind its prevalence because it already ascribes to the norm. As Llano states: “In many other countries the oral exam isn’t just normal, it’s expected” (2021). Although the classical forefathers preferred oral communication and shared a “love-hate relationship with writing, fearing its erosion of people’s ability to remember” (Heinrichs, 1995, p. 39), rhetorical tradition has also had to long vie for its stance in the academic world. Heinrichs laments the fall of the original value of the rhetorical tradition to the proclamatory role of religion and “modern science” (39); Descartes’ rejection of probability and the repudiation of “contingent truths” transformed rhetoric from “oral persuasion” and “consensus building” into “the proliferation of jargon” and “an art of showing off” (pp. 39-40). Seen in this light, we begin to understand how the written form became the accepted “choice” of performance measurement for many in the academic world (40).

A fair discussion of the philosophical underpinnings of rhetoric clearly goes beyond the scope of this paper; however, understanding the main cultural/linguistic influences of the past may help us better understand the dichotomies concerning oral communication present today in higher education: i.e. an established tradition of oral assessment in Euro-
pean institutions of higher education but a lack of explicit training for students to improve their academic oral communication skills. We live in a world in which effective oral communication is more important than ever due to increasing globalization, including the risk that oral communication may someday become secondary to written communication via social media. As Donskikh explains: “...the primacy of oral communication is its dialogic nature, which is radically different from our primarily monologic reality. Involvement in the subject was being organized through dialogue, not through abstract acquaintance with it, especially with the usage of written text” (8).

It is surprising then, that a broader, more explicit treatment of oral communication does not exist in university-level academic settings. This situation is not only apparent in European education, as Heinrichs describes the academic program at Harvard University, where “In the midst of an increasingly oracular society, the College requires that its students study expository writing (and has done so since 1872), but doesn’t offer a single course in which oratorical theory and practice are taught together. As the ancients understood, the two are inseparable” (p. 42).

Like Heinrichs, other voices have been calling for a more balanced treatment of oral and written discourse in higher education. Here, both written and oral assessment of students’ knowledge of a topic is “wholistic” (Llano, 2021) and a combination of both forms is seen as the “best” for students (Paulsen, 1966, p. 427; see also Kehm, 2001, p. 27). Joughin pleads for a “balanced diet” in terms of assessment methods (Joughin, 2010, p. 1).

Indeed, oral assessment clearly has strengths in its own right. The discursive nature of oral assessments is seen as the best way to evaluate students’ critical thinking about an issue (Kehm, 2001, p. 28); oral exams confront students with challenging questions that test the depth and scope of their knowledge, a situation they may likely be confronted with as professionals (Ehrlich, 2007, p. 375). Applauded is the “probing” nature of questions asked in oral exams (Theobold, 2021, p. 1; Ehrlich,
2007, p. 375; Dicks et al., 2012, p. 1506). Oral exams simulate real-world situations and are deemed as more authentic (Theobold, 2021, p. 1). In addition, they can encourage meaningful discussions between students and faculty members (Dicks et al., 2012, p. 1506). Oral examinations are unique in that they enable instructors to evaluate several skills, such as critical thinking, presentation skills and “gauge” these in “real time” (p. 1509).

Nonetheless there are still non-philosophical reasons for not using mandatory oral assessments, including factors such as lack of time, a perceived lack of objectivity, as well as potential legal issues. Many professors see oral examinations as too time-consuming (Buis, 2021). Especially with larger classes, instructors complain that they do not have enough time to administer so many oral examinations (Dicks et al., 2012, p. 1506). Another potential disadvantage of the oral exam is the worry about lack of impartiality on the part of the examiners. Here the proctor may not be completely objective in his/her evaluation of student performance. Depending on the political arena, oral examinations can even be seen as “oppressive instruments of the state” (Kehm, 2001, p. 28). Lastly, in some academic settings, legal concerns are an issue and may deter university administration from wanting to administer oral examinations. As an instructor from the United States explains, students may feel that they are being treated unfairly and take legal action to contest a grade they see as unfair (Llano, 2021; see also Joughin, 2010, p. 7). Referring to the situation in the United States, Theobold states that the “…‘standard’ assessment diet in collegiate STEM courses consists of written, timed exams” (Theobold, 2021, p. 1).

3 Conclusion to section on history of oral communication in European education

Despite the long tradition of oral communication in European education and its fixed role in institutional assessment structures, questions remain. While most students are assessed orally, there is little evidence of explicit training in oral communication in the higher-level academic
setting, that is, in the seminar, in the lecture, or in group discussions. Students are often left to their own devices when it comes to systematic preparation, not only for oral assessment situations but for everyday content-based oral communication in a university context (Briggs et al., 2012; Thomas et al., 2012). In particular, this pertains to Bachelor’s students who have just completed their high school education and are beginning their first university semester.

What lies behind the tacit assumption that students require no outward preparation may relate in part to the stance that oral communication is inferior to or less important than the production of written academic texts. In fact, many professors value written texts alone as evidence of student comprehension of subject matter. When asked about how to help students who may encounter difficulty in speaking about their content matter with colleagues and with professors, one professor’s response to the author was: “As long as they can write well that’s fine” (author’s personal communication, April 29, 2021).

Feak gives us plausible practical (not philosophical or rhetorical) reasons in the past for the predominant emphasis on written language in higher education. For one, accessibility to written texts was much easier (2014, p. 34). However, this situation has changed dramatically with the widespread use of smart phones and hand held devices for the recording and collecting of, as well as access to authentic texts (see also Hughes et al., 2010). In addition, online learning platforms (e.g. Moodle) enable learners to record texts with great ease. Yet another area is the massive collection of authentic spoken texts in various corpora in various languages. As Hughes notes, spoken data can literally be collected anywhere these days (Hughes, 2010).

In many ways, students must acquire content-specific oral communication skills on their own, not only before they enter university but also during their studies and leading up to their formative oral assessment exams. This situation is even more intensified for international students as they begin their university studies. On the one hand they must demonstrate the required language level for university studies (usually
B2/C1 level, depending on the program of study) and arrive at university confident that they have the language skills necessary to begin a program of study in a language not their L1. However, to their dismay, especially in the first few weeks, these students are confronted with a high level of abstraction in lecture and seminar discussions, a situation much different from the more formalized and structured discussions and conversations they have encountered in their classes in high school.

4 Scientific discourse

“… the mastery of a specialized subject is in large part mastery of its specialized ways of using language” (Lemke, 1990, p. 21). Oral discourse is central to a learner’s understanding of content-related meaning and its application. According to Lemke, by “talking science”, we are able to “do science”, because science is embedded in the interaction amongst its members and is clearly a “social process” “where we talk to ourselves and with others” and “create, communicate, and use knowledge” (pp. ix-xii).

Key to scientific discourse is identifying and recognizing the content-related elements as they are received in the communicative exchange and then knowing when and in which way to produce them meaningfully with others. The goal here is to be able to decipher what is content-related, a skill which must be learned (Lemke, 1990, p. 12). In other words, students must learn how to “extract the science meaning” from the dialogue in the classroom (p. 12).

Perhaps the greatest error of equating oral proficiency directly to written proficiency is the false notion that spoken text is merely an audible version of written text. As Halliday tells us, “people do not actually speak in sentences and paragraphs” (1989, p. 115).

Spoken language is spontaneous, not planned. It is simply false that spoken language is merely comprised of turn-taking. Basturkmen argues that meaning is co-constructed (interlocutors often speak at the same
Two main types of content-related dialogue take place in the classroom – teacher-student and peer-peer communication. Common to teacher-student interaction is the ‘triadic dialogue’ (Lemke, 1990, p. 19), in which the teacher initiates discourse, the student responds, after which the teacher gives the student feedback on his/her response (Salloum & BouJaounde, 2017, p. 7). If the rules for dialogic exchange remain too pre-determined, it may negatively affect a student’s own explicit learning process. Central to meaningful scientific dialogue is the negotiation of meaning through discourse. In teacher-student interaction, this can take place in referential questions, where both interlocutors “genuinely” seek new information (Lynch, 1992, p. 110). The potential results of this type of dialogue sound promising for several reasons: student answers are typically longer, students are more likely to show more active participation in the dialogue as the questions are less one-sided than display questions (p. 110). “It [oral communication] forces students to highlight main points, to keep the thread of the narrative, turning a monologue of the teacher into a dialogue between teacher and student” (Donskikh, 2019, p. 11). According to Lynch, referential questions can play an important role in the negotiation of meaning. Teachers should be encouraged to incorporate this type of question in classroom instruction, because it can “create opportunities for interaction that allow freer language use” (1992, p. 110). Spoken interaction on a learner to learner basis is more likely to lead to negotiation of meaning than teacher to learner interaction (see Doughty & Pica, 1986).

Important side-benefits of negotiation talks are that students can gain cognitively from these experiences – cognitive experiences help students to learn better. Students also profit on an affective level from the social interaction with their peers (Lynch, 1992, p.113).

Lemke also mentions the use of metadiscourse (1990, p. 118) and explains that this strategy or type of discourse can be used for decoding purposes. However, this could also relate to what Lemke refers to as
**reasoning** (global strategy) (p. 121) and moreover “what makes reasoning logical is that it follows certain rhetorical and genre structure patterns” (p. 122).

Lemke makes a plea for changes in science education and notes that paying greater attention to language skills is key to student success. In his opinion, students need to practice different communicative situations (dialogue and monologue) and have more time and for a longer period of time (at length) to practice “speaking” (1990, p. 169).

### 5 CLIL, semantic relationships and didactic progression

In addition to ample time to speak science in the classroom, teachers need to provide students with the time to learn and to be using key content-related semantic items that students need to negotiate meaning.

“…making sense means identifying the semantic relationships between the words and phrases used, that is, hearing them in the context of a thematic pattern”

(Lemke, 1990, p. 92)

“Talking science, in the fullest sense, always combines a thematic pattern of semantic relationships with a structural pattern for organizing how we will express (i.e. construct) them”

(Lemke, 1990, p. 123)

Once in possession of these semantic items, students would first begin to use simple sentences and then progress to the use of complex sentences – where they will be able to combine several semantic relations in one sentence (Lemke, 169). Lemke goes on to suggest a particular didactic order for the practice of content-related oral communication (p. 169). First, science terms should be combined in longer grammatical sentences. Students should describe, compare or discuss real objects or events using the science terms in a flexible manner appropriate to the situation. Then, these terms can be further used to write sentences and paragraphs that have been derived directly from the oral discussion.
Lemke sees oral communication as the basis and not just as an alternative to the goal of writing proficiency.

Teachers model scientific language and explain how to combine terms in sentences (see also Khisty & Chval, 2002). Content-specific idioms and phrases, as well as particular grammatical constructions are explained. Semantic relations of terms and various ways of expressing the same relationship in different words are discussed and practiced.

Language instructors need to explicitly identify when two expressions with the same meaning might be used appropriately in constrasting registers, i.e. formal vs. informal/colloquial and/or spoken vs. written. According to Lemke, “Students should regularly have oral and occasionally written practice in class in restating scientific expressions in their own colloquial words and also in translating colloquial arguments into formal scientific language” (1990, p. 173). Buhlmann and Fears suggest a similar lexical progression, beginning at the word level, then proceeding to sentence level and finally engaging on a textual/discourse level (2019, p. 303).

6 Enter Mathematics

The apparent dichotomy in the situation described above has its roots in content. Despite a high general language proficiency, new students are often not able to fully participate in the content-related discourse because “What is being said may not fit any thematic pattern they are used to” (Lemke, 1990, p. 27). In other words, students may have received general adequate language training but were not given enough language training in content areas such as mathematics.

Learning to talk science differs from learning how to speak in a foreign language (Lemke, p. 160). While in foreign language learning the topics are familiar and the grammar may be new, the language of mathematics is the opposite case, that is, the grammar may be familiar while the themes are new (p. 160).
Mathematics is the common denominator for countless academic and professional fields, yet its importance and its relevance are often underestimated (Kerstan, 2021) as students arrive in their first semester often unprepared for the math content, and corresponding language, that awaits them. According to a recent study, professors in Germany perceive today’s students as not fit enough “zu dumm” to study at university (reported in Kerstan, 2021). The reason for this may in part be attributed to a lack of transparency between schools and universities (Kerstan, 2021).

Kisty and Chval pose the question: “Where does the social language of mathematics come from? Since it is a specialized discourse that would not readily be heard in most social contexts, we can assume that it is not acquired in the same way as everyday language” (2002, p. 156). Although it is “applied” worldwide, contrary to popular belief, mathematics is not a universal language and does not just differ slightly from general academic or scientific language. The language of mathematics comprises more than just “minimal linguistic challenges” (Fernandes, 2012, p. 10), where “good teaching” alone may suffice (p. 10). Aspects such as register, lexical level, semiotic traps, as well as grammar issues, pose challenges to students as they learn the specific academic language of mathematics (pp. 10-11).

Implicit oral communication in mathematics takes place for students in various communicative situations in the university setting. This includes lectures, where they may answer questions posed by the professor, or in semi-formal conversations or tutorials which accompany the lectures. Tutorials are led by upper-undergraduate and graduate students, meet on a weekly basis and give students oral and written (“chalk talk”) feedback on their homework (see Artemeva & Fox, 2011). During the tutorial, students also meet in small working groups (2-4 persons) where they discuss how to (best) prove mathematical problems (based on themes from most recent lecture). The tutor provides students with referential instead of direct answers to questions pertaining to homework problems. In this way students have an idea in which direction they could further (or better) prove the mathematical problem; however,
they have to perform the mathematical “legwork” themselves. Other situations in which students would “speak” mathematics include informal study groups, similar to discussions that take place in working groups in the tutorial (and may be the same participants). Lastly, students may attend a professor’s or tutor’s office hours and engage in a one-on-one dialogue.

What value does this oral communication have for these students in their studies? Or rather, what could a lack of content-related oral communication skills mean for students? A number of aspects must be considered. Firstly, when students do not engage in a dialogue with fellow students, they socially isolate themselves. Secondly, students isolate themselves in their own content knowledge – a lack of peer exchange could prevent them from reaching new levels of conceptual understanding. Lastly, little or no practice of semi-formal oral communication necessary for exit examinations could directly affect success in earning a university degree.

With these communicative goals in mind, a more meaningful and authentic content-related discourse requires an explicit approach towards the extraction of thematic patterns directly related to the particular content (Lemke, 1990, p. 22). This can take place in the content-specific language classroom. Concretely, this means that whether “students become proficient in mathematics depends on understanding the processes of interaction and the characteristics of talk in this classroom” (Kisty & Chval, p. 154).

The lexical aspect of communication and the emphasis on the importance of creating a learning environment means seeding the teaching environment with “rich words” (Kisty & Chval, p. 155). Furthermore, these ‘rich’ elements need to be given their own meaning and context in order to be internalized by the learner (Kisty & Chval, p. 155).
7 Motivation behind the course

One-fifth of the student body at the Technical University of Darmstadt is comprised of international students, that is, students whose home country is not Germany and whose first language (L1) is not German. With the exception of the Department of History and Humanities, all degree programs at TU Darmstadt require courses in mathematics as a core element in the curriculum. Concretely, this means that 90% of all students need to understand and communicate “mathematically”; nevertheless, the actual broader relevance of mathematics for academic and professional purposes is in large part underestimated, especially in non-MINT-related fields (Kerstan, 2021).

While mathematics is on the one hand a common communicative denominator for most students at a technical university, it is also a source of frustration and one major reason for international students to drop out of university. Students are required to have at least an advanced intermediate language proficiency level (B2/C1 European Language Reference) in order to be admitted to a course of study; however, this proficiency level as well as the tests designed to assess this level do not measure a student’s ability to perform in a content-specific communication situations. Such situations could include listening comprehension and effective note-taking of a lecture on thermodynamics, or a discussion with peers on how to prove certain mathematical statements, for example.

8 Learners

With this in mind, a content-based language course in mathematics was developed for international Bachelor’s students starting their degree at a technical university in Germany. The learner group was heterogeneous, with students coming from all areas of the world. The major areas represented included: Syria, Tunesia, South Korea, Iraq, China, Turkey, and India. These students were pursuing a first degree either in natural sciences or engineering and were required to take introductory
math classes in the first 1-2 semesters of their studies. Mathematics is the language they use to discuss pragmatic engineering solutions. Math is the lens through which they can precisely describe the nature, the complexity and the viability of the engineering problems they would be confronted with. With this in mind, structuring a language for special purposes course around the language of mathematics was sure to reach a diverse group of students.

In the initial two years (2017 & 2018), one language course was offered. Between 2018 and 2019 the number of participants doubled, and this number doubled again so that in 2019 two German CLIL language courses were established to meet the growing number of interested students. In 2020 the number of enrollments remained stable so that two courses were again offered to a total of 46 students.

9 Course structure

The two-week intensive German language course included 36 teaching hours and took place in the month of September (pre-semester), so that students had time to acclimate themselves in their new homes as well as new learning environments before the semester began in early October. After attending the language course, most students would go on to take part in an elective (non-mandatory) mathematics course in the latter part of September. In this way, the content-based language course in mathematics was designed to prepare students not only for a smooth transition to the beginning of the semester, but also to give them immediate practise in the pre-semester math course.¹

The primary goal of the language course is to improve students' content-related oral proficiency in communicating mathematics in a monologic and dialogic sense. The courses were planned and led by language instructors, one of whom has ten years of experience in language for special purposes (German for Engineers and English for Engineers), the

¹ The German language course is part of the PreCIS study preparatory program and includes further elements such as: study skills training, campus orientation, as well as workshop and buddy programs offered during the semester.
other who has seven years in German and English language teaching as well as 28 years of experience in the translation of technical texts.

The course structure has changed or rather evolved over time, based in large part on the feedback received from student evaluations. After its second year in session (2018), students expressed a clear wish for even more content-based feedback during instruction.

In order to implement these changes within the language course, two significant steps were taken. Firstly, the author established direct communication with the university’s department of mathematics. Questions for an interview were prepared and the author met with a mathematics professor who also had taught introductory mathematics courses (first semester mathematics) for many years. He kindly shared in detail his lecture notes and homework assignments from the past. In addition, based on the interview questions, he was able to describe the communicative settings where mathematics was being “spoken”, namely during the lecture, tutorial sessions, office hours or in student study groups. This detailed account helped the author better understand how mathematical language was represented/used by the various participants in the various academic settings.

Separate meetings with Ph.D. students in the mathematics department (who were also in part involved in teaching the introductory mathematics lectures or who had served as tutors in mandatory tutorial sessions corresponding with the mathematics lecture) also provided invaluable input for changes in our language course structure. The author wanted to know which mathematical symbols were most important for first-semester students attending the Mathematics 1 lecture. Prior to the meetings, the author had compiled a list of potential mathematical symbols based on readings and prior research. The colleagues in the math department were able to confirm the importance of most of these terms, as well as remove and add a few terms. In all, almost 60 symbols comprised this list (see Appendix 1). The way in which these terms were put into practice during language instruction will be explained at a later point in this paper.
Further discussions concerning this list of symbols centered around concrete differences in register between various communicative situations students would encounter. The author learned that for most symbols there exist varying degrees of (in)formality. For example, the formal use of a mathematical symbol may be used by a professor formally in a lecture, however, the monologue in which these symbols appear could very well contain informal ways of expressing mathematical expressions.

Another factor that became apparent in these discussions was the presence of ideolects in the verbalization of mathematical symbols and mathematical phrases. That is, that there are various ways learners can “speak” mathematics. For this reason, the author, as well as most language instructors, would not be able to accurately say whether a mathematical variation was correctly expressed or not.

In the next step, the language instructors requested that the colleagues in the mathematics department compile a list of 20 mathematical statements (see Appendix 2) in which most of the 60 symbols from the list were represented. The audio recordings of these mathematical statements also reflected ideolects, so that for each sentence there were two to three “correct” variations. In addition, the language instructor made sure that male and female voices were equally represented in the recordings.

10 Implementation of highly specific content-based material in the language classroom

With the help of valuable lexical content input from the mathematics department, the language course acquired a much stronger emphasis on lexis. These lexical units (60 symbols and 20 mathematical statements) were first and foremost developed for oral communication tasks to be practiced in the language classroom. The lexical progression within the two-week intensive course took place in three phases: word level, sentence level and finally, textual level.
At the beginning of the course students were made aware of the communicative dichotomy they find themselves in. That is, on the one hand they were clearly able to read (to themselves) and understand most of the individual mathematical symbols they are presented with at the beginning of the course. On the other hand, when asked to read the symbols out loud or rather read a combination of symbols in the form of a mathematical sentence, they quickly recognized the presence of a language barrier, namely the inability to verbalize this information themselves or to others. As these students are quite proud of achieving a language level high enough to attend university, they were initially surprised at this obvious hurdle. For language instructors this hurdle is not a curse but actually a blessing, because it serves as a primary motivator for students throughout the course, as they know that they will soon be needing this knowledge to participate in mathematical discussions with peers as well as with professors. The realization here for students is twofold: firstly, they recognized that they have received little or insufficient explicit content-based training in mathematics prior to entering university; secondly, students realized that despite their high level of language competency, without oral competency, “speaking” mathematics quickly becomes a challenge.

With the aid of sentences containing most of the 60 symbols provided by the mathematics department, students were then able to combine and reproduce their lexical knowledge in bigger chunks. On a sentence level and beyond, the language of mathematics allows for individual variation or idiolects. One could say that each student “speaks” mathematics differently. It would be fair to say that clear knowledge of such variations, as well as when and why certain variations are permissible transcend the scope and training of most language instructors. With the introduction of content-related student tutors to the language classroom, such questions could be answered meaningfully for students. In turn, language instructors were able to focus more on language issues while knowing that content-related questions were being addressed in a structured and meaningful way by the tutors.
The second major change to the language course was the implementation of content-based math tutors in September 2019. These tutors were sought out and hired by the TU Centre for Teacher Education (Zentrum für Lehrerbildung). Tutors already had several semesters of university experience, and most tutors had already had some light teaching experience (tutoring, internship at a school). The tutoring position is intended primarily for future schoolteachers and is one of several practice-based options students can choose from in the mandatory second phase of practical experience in their course of study. Prior to their work as tutors in the language course, these students take part in an intercultural seminar where they are made aware of and reflect on potential language and intercultural issues they may be confronted with as a tutor. These issues are ones which these students deal with later on as full-fledged teachers in their own classrooms, namely, intercultural communication and the difference between general language and language for specific purposes.

The tutors’ depth of content-based mathematical knowledge as well as their experience using that knowledge in their studies were an asset to course development. Based on their academic background, the tutors engaged with students differently. Students benefitted from various forms of feedback.

In general, just as students may gain from the authentic content-related feedback from tutors, tutors also learn more about how language can help or hinder content-related learning. It is important that pre-service teachers are made aware of the influence of language on content-related teaching, especially in a heterogeneous setting where students may have a different L1 than the language being spoken in the classroom. More importantly, it is important that pre-service teachers recognize that competency in everyday language does not suffice or automatically mean that students have the same level of proficiency in academic language, in this case the language of mathematics (Fernandes, 2012, p. 18). As Eggins states: “Knowledge of the basics of systemic functional
linguistics is also essential in understanding the linguistic complexity
in the formulation of problems and how this may impact ELL students' communication of their mathematical thinking” (2004, p. 19).

The first year with tutors in the language classroom included one teacher in training (subject areas: biology, ethics and chemistry) and one Master’s student in mechanical engineering. The year thereafter both tutors were pre-service mathematics teachers. Each tutor brought valuable knowledge and experience to the classroom. Some were further along in their studies than others; some had more prior teaching experience than others. The engineer approached mathematics from a more practical view as this was the type of math he used in his studies. The pre-service teachers had more knowledge of pedagogical aspects such as lesson planning and strategies for giving meaningful feedback on assignments. The pre-service teachers were also interested in playing an active role in developing assessment tools planned for the final exam in the course.

Tutors were actively involved in course planning prior to the course beginning. They were responsible for preparing various content-related tasks for language instruction, including additional audio recordings of mathematical phrases from the math department. They performed audio recordings of materials that were only available in written text form. Each recording was different, based on each tutor’s individual mathematical idiolect. Tutors also made an important contribution toward finding a variety of authentic examples for exercises involving text level language.

During language instruction, tutors answered questions about variations in the names of the particular symbols (word level). They were also able to illustrate examples of register, that is, that a more formal level of reference to symbols takes place in written texts and in lectures; but that during tutorials the tutor’s language, as well as discussions amongst peers, could be informal. The author also gathered this impression firsthand after observing several sessions of a math tutorial and the interactions between the tutor and students, as well as amongst
the students themselves.

Course tasks on the third, namely text, level reached a much higher degree of content-related specificity and authenticity based on the knowledge and situative experience of the tutors. Here tutors played a central role in designing and implementing the teaching units that took place on the text level.

The interplay between language instructor and tutor is a highly useful one for several reasons. Firstly, the tutors bring a high level of content-related knowledge of mathematics to the language classroom. The selection of materials, especially those on a textual level, enabled language instructors to provide a greater level of content specificity in language-related tasks, which students were able to learn from in the classroom and beyond. Tutors can estimate the relevancy of the materials based on the prior experience they have already had in such mathematics courses. The language teacher can use the materials created by tutors to develop language-based materials for the classroom. Language teachers can develop activities based more on the language of mathematics itself and less on the mathematical content.

In turn, in the planning phase tutors are exposed to the approach of the language instructor. Tutors learn about the typical characteristics of mathematics as a language, i.e. this language needs to be learned explicitly and that it differs from a general knowledge and standard use of German. Next to English, German is the second most commonly used CLIL language (Feak, 2014, p. 40).

Language instructors and student tutors benefit from one another in this setting, in part because both are justified in their language use in their own right and area of expertise. While on the one hand “…content or discipline specialists have limited knowledge of how second languages are learned, [language teachers] have the limited ability to anticipate or replicate how teaching is accomplished in the various disciplines” (Gorsuch, 2006, p. 91). To illustrate this, we can refer to Table 1, and see how the roles of language teachers and tutors complement one another in a
CLIL setting as students learn how to prove a complex mathematical problem (text level).

When introduced to a new text, language learners should be able to read the text out loud so that it is audibly understandable (intonation, pronunciation). Should the students not understand certain words in the text, they should be able to use certain decoding strategies particular to the language of mathematics (word form, compound nouns, false friends\(^2\)) to help them decipher meaning. While the language teacher will focus on these goals for students, the tutor will be able to clarify content-related questions pertaining to unknown terms in the text. The tutor may also offer potential variations in reading the text, especially related to the reading of mathematical sentences.

Using the mathematical text provided by the tutor, the language teacher can develop exercises related to the practice of specific mathematical vocabulary terms as well as general structuring or orienting academic words which may structure the mathematical text. Listening exercises that ask students to complete the text and anticipate meaning will help students train their ability to read, understand and speak about longer mathematical texts with their peers. Parallel to this, the tutor would model the proof of the mathematical problem on the chalkboard (or with the use of digital tools in an online course). Here students are confronted with a variety of useful and authentic aspects, as they will encounter this same situation in the tutorial accompanying the Math I lecture. By observing the tutor, students see how proofs are written. As the tutor writes the individual steps of the proof on the blackboard i.e. screen and explains these orally (use of metadiscourse), s/he assigns a name to what is being done in each of these steps with the use of content-related performative verbs such as factorize, derive, etc. This process of scaffolding provided by the tutor directly and more indirectly by the language teacher (in the form of in-class exercises) helps students to create their own written and oral mathematical texts.

As the tutor was scaffolding the oral text production on the board, the

\(^2\) Words that are orthographically identical but on a semantical level different.
author was able to observe students’ particular interest in the performative verbs. As the tutor was explaining, students were busy taking notes on what was being said i.e. demonstrated. In a later exchange with the author, the tutor mentioned that he hid his surprise that the students had taken such detailed notes on a mathematical theme that should not have been new to them. The author responded to the tutor that the students were very interested in hearing the names assigned to the various steps involved in explaining a mathematical proof.

After an initial reception of the text, all the students are given a similar, but new text and are asked to prove the mathematical problem. The mathematical problem poses no new mathematical content to the students; however, the level is high enough to bridge experience in the language course with later mathematics courses. As students work on their own in pairs to solve the mathematical problem, many students could be observed implementing the steps modelled by the tutor and in the materials developed by the language teacher: reading the text out loud to one another; marking unknown content words (especially compound nouns) and defining these; marking signal words key to text comprehension; using newly-learned performative verbs to explain the various steps involved in their proofs. Unlike the monologues heard in the modelling phase, students now engage in a dialogue and negotiate with one another how the proof should be solved. Students’ oral presentations of the results (combined with written text on the board) are met with feedback from the language instructor and tutor. The language instructor focuses on correct use of syntax (word order), word forms, use of structuring terms and pronunciation of mathematical symbols, whereas the tutor will give feedback on the efficiency of the proof, that is, whether a shorter explanation may be possible as well as possible variations on how to read the mathematical sentences (ideolects). Because students have all been given the same text, they are able to compare and contrast their answers with one another.

In a further phase, student pairs are each given a separate mathematical text that needs to be proved. The presentation of their proof serves as the oral portion of the final exam for the course. After the presentation,
**TABLE 1:** Proving mathematical texts – how language instructors and content-specific tutors complement one another in a CLIL setting

<table>
<thead>
<tr>
<th>Role(s) of language instructor</th>
<th>Role(s) of content-specific tutors</th>
<th>Type of Oral Communication</th>
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</thead>
<tbody>
<tr>
<td><strong>Reception of Text</strong></td>
<td>Focus on certain content-related items</td>
<td>Monologue</td>
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<tr>
<td>Read text out loud</td>
<td>Offer related examples</td>
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<tr>
<td>Practice intonation</td>
<td>Discuss potential variations (ideolects) (while reading out loud)</td>
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<tr>
<td>Reading and decoding strategies</td>
<td></td>
<td></td>
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<tr>
<td>Identify LSP elements</td>
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<td></td>
</tr>
<tr>
<td>- compound nouns</td>
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<tr>
<td>- structuring words</td>
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</tr>
<tr>
<td>false friends</td>
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</tr>
<tr>
<td><strong>Analysis of Oral Text Production</strong></td>
<td>Write proof on board</td>
<td>Expert-novice dialogue (question and answer)</td>
</tr>
<tr>
<td>Listening texts to authentic recordings of how math sentences are proved.</td>
<td>- „chalk talk“ in mathematical context</td>
<td></td>
</tr>
<tr>
<td>- Practice of lexical terms</td>
<td>Metadiscourse of steps</td>
<td></td>
</tr>
<tr>
<td>Review of structure marking words</td>
<td>- Use of „we“</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- How to negotiation meaning</td>
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</tr>
<tr>
<td></td>
<td>Performative verbs</td>
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<tr>
<td><strong>Oral text production via scaffolding (same text for all) in pairs</strong></td>
<td>Feedback on:</td>
<td>Peer-to-peer dialogue</td>
</tr>
<tr>
<td>Feedback on:</td>
<td>- Sentence structure</td>
<td>Tutor-peer dialogue</td>
</tr>
<tr>
<td></td>
<td>- Structure-marking words</td>
<td></td>
</tr>
<tr>
<td>Use of academic vocabulary</td>
<td>Feedback:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Content-based</td>
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</tr>
<tr>
<td></td>
<td>Confirmation of variations (ideolects)</td>
<td></td>
</tr>
<tr>
<td><strong>Free oral text production (pairs)</strong></td>
<td>Feedback on:</td>
<td>Peer-to-peer dialogue</td>
</tr>
<tr>
<td>Feedback on:</td>
<td>- Sentence structure</td>
<td>Tutor-peer-dialogue</td>
</tr>
<tr>
<td></td>
<td>- Structure-marking words</td>
<td></td>
</tr>
<tr>
<td>Use of academic vocabulary</td>
<td>Feedback:</td>
<td></td>
</tr>
<tr>
<td>- Confirmation of variations in academic vocabulary</td>
<td>- Content-based</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Confirmation of variations (ideolects)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Suggestions for greater efficiency in &quot;speaking math&quot;</td>
<td></td>
</tr>
</tbody>
</table>
the students who have presented receive written feedback from the language teacher (focus: language issues such as pronunciation, syntax, use of vocabulary, etc.) from the tutor (validity and efficiency of mathematical content, as well as accuracy of spoken mathematical language) and from peers (clarity, main strengths and weaknesses).

12 Student evaluation results

Student opinions of oral assessment and of oral communication as a part of learning in general gives us a much clearer and more optimistic picture of the learning process. In general students prefer language instruction that is: “interactive, real-life focused, open dialogue and feedback based teaching model” (Segal & Vasilache, 2012, p. 525).

Evaluation results (2020) indicate a high level of satisfaction among students attending the content-related language course for mathematics. After having attended the language course, they felt better prepared for mathematics lectures and tutorials (Übungen) in their studies; they were highly satisfied with the team of language instructor and tutor (Seibel & Yapakci, 2021, p. 2).

Individual comments which highlight student satisfaction with the course include (pp. 3-4):

“Für mich war der Sprachkurs hilfreich, weil ich meine Kenntnisse in der Fachsprache Mathematik erweitert habe.“ [The language course was helpful because I increased my knowledge in the language of mathematics.]

“Deutschkurs für Mathematik - für die Uni nützlich, die Begriffe in Mathematik wären zu einer großen Hürde geworden.” [German course for mathematics – useful for the university, the mathematical terms would have been a big hurdle otherwise.]

“Mir hat der Sprachkurs sehr gefallen, da ich viele neue Wörter gelernt
“Lernen von mathematischen Begriffen” [Learning new mathematical terms]

“Fachsprache Mathematik, weil ich vor dem Kurs keine Mathe Wortschatz hatte.” [Language course in mathematics, because prior to the course I had no mathematical vocabulary.]

“Die Fachsprache Mathematik…” [Specialized language of mathematics]

Students mention an increase in knowledge and use of the specialized language of mathematics as being the most important benefits of the language course.

The fact that students refer to the language of mathematics and its specialized terms (lexic) in their comments shows that they have developed a clear awareness that a separate language of mathematics needs to be learned, practiced and handled with care.

13 Response of tutors

The responses below highlight one tutor’s reflection on her previous work in the content-related language course for mathematics. This student is studying mathematics and philosophy and is in her fourth semester of her Bachelor’s degree. She has worked as a CLIL tutor for the past three consecutive semesters.

Similar to the student evaluations, the tutor also views vocabulary as critical for student progress.

“Vokabeln (sind) ein großer Teil des Lernzuwachses und natürlich Grundlagen, um überhaupt über mathematische Gegenstände sprechen
zu können. Glücklicherweise gibt es eine präzise Formelsprache in der Mathematik."

[Vocabulary is a major part of learning and, of course, the basis for being able to talk about mathematical subjects at all. Fortunately, there is a precise formula language in mathematics.]

The tutor is able to reflect on how learning takes place in the classroom and explained that mathematical terminology has to be learned by all students, including native speakers.

"Die Benennung von Symbolen und die Hin- und Rückübersetzung logischer Ausdrücke in deutsche Sätze ist ein wichtiger Teil des Erlernens mathematischer Fachsprache, das gilt auch für Muttersprachler."

[Naming symbols and translating logical expressions back and forth into German sentences is an important part of learning mathematical terminology, and this is true even for native speakers.]

In addition to the learning and practice of key mathematical terms, a general command of presentation skills comprises rhetorical competency and not merely content-specific knowledge.

"Im Kurs üben wir die Formulierung einer Aufgabe/Frage und die Benennung von Rechenschritten. Der Vortrag erfordert aber vor allem, die Gedanken klar und strukturiert zu vermitteln."

[In the course we practice formulating a task/question and naming calculation steps. However, the presentation requires above all the ability to convey thoughts in a clear and structured way.]

She noticed how knowledge of word formation (morphology) can be used strategically in speaking mathematically.

"Teile desselben Gebildes haben oft gemeinsame Wortbausteine, zum Beispiel kommt in einer „Gleichung“ das „Gleichheitszeichen“ vor,
FLEISCHHAUER
das als „ist gleich“ vorgelesen wird oder bei einer „Division“ teilt man „Dividend“ durch „Divisor:“

[Parts of the same entity often have common word components.]

Language can positively affect or inhibit students when learning new mathematical content. The tutor observed the following in class:

“Wenn Kurteilnehmende versuchen, etwas zu erklären, dessen Namen sie nicht kennen, verwenden sie (vielleicht nur aus Versehen) Komposita, um uns mit wenigen Worten, aber trotzdem präzise, in die richtige Richtung zu lenken. Wenn sie feststellen, dass ein vermeindlich ausgedachtes Wort tatsächlich genau so oder so ähnlich bekannt ist und genutzt wird, sind die meisten überrascht.”

[When course participants try to explain something they don’t know the name of, they use composites (perhaps only by mistake) to steer us in the right direction with few words, but nevertheless precisely. When they discover that a supposedly made-up word is actually known and used in exactly the same or similar way, most are surprised.]

The tutor also noted how she has incorporated the teaching methods she observed the language instructor using in her own role as a tutor:


[What XXX does and what I have come to appreciate after some initial confusion is to start a casual conversation with the participants and ask]
semi-personal questions. In a conversation about moving to Germany, for example, you’ll casually encounter topics such as distances, time difference, date pronunciation, and currencies. And the ability to use numbers in everyday life cannot be overestimated. That’s why in my methodology I’m moving more and more in the direction of looking at a small event as the starting point for a potentially wide-ranging conversation. Because in a language course, nothing is more important than talking.]

14 Response from one language instructor to the role of content-related tutors in CLIL setting

One of the key advantages of CLIL instruction is generally that „language learning is no longer predominantly perceived as a boring and tedious chore but as a communication tool that can be mastered and honed to access knowledge in a field of high personal interest. Students prefer learning situations that mirror real-life scenarios!”

Tutors enhance the course at several levels and serve as “student role models”. They play a mediating role in the CLIL setting by filling “a knowledge gap” and acting as “go-between communicators” (mediators) between language teachers and language learners.

“Tutors also communicate as subject experts with language-learning students because a) they can answer subject-specific questions in greater depth, b) they can provide instant feedback on correctly used expert language even if student answers differ from standard expectations, c) they pave the way for higher flexibility in how students may respond with regard to content, solution methods, expert vocabulary and phrasing.”

Tutors are “living proof that success is possible” and can point out resources on campus that helped them in their studies. They can offer students first-hand feedback on how to communicate with different actors at the university: professors, university teachers and fellow students.
While language teachers provide the linguistic structuring and foundation, it is the interaction with tutors that helps to put the newly learned skills and information into perspective.

“Tutors in CLIL courses learn that a) being an expert is not enough to communicate knowledge, facts and processes successfully, b) not being able to communicate adequately and successfully isn’t a sign of not being an expert, c) methods used to communicate knowledge, facts and processes may differ in various cultures, d) knowing the target group and culture is essential for successful communication, e) accepting variations in register, accent and style while treating communication counterparts respectfully as equals facilitates communication and understanding.”

15 Conclusion

“The key to success in mathematics is being able to own the language and concepts that make mathematical language different from ordinary English”

(Jourdain & Sharma, 2016, p. 52)

The oral tradition in European higher education will continue as will the need for graduates to communicate effectively, precisely, and critically in a global marketplace that continues to increase in importance. It is clear that not simply assessing but training students in content-based oral communication situations needs to be treated more explicitly in the future. From a societal perspective, it is important to keep the tradition of discourse alive, especially in an age when we risk communicating more through the medium of writing with the help of our devices and via platforms than actually with one another.

Our CLIL language course experiences have demonstrated that content-specific language courses are a viable solution for students as well as for university administration. Providing students with adequate as well as authentic content is a visible challenge for language instructors,
as they are linguists and normally not specialists in the natural sciences or mathematics. The integration of student CLIL tutors has proven to be an enormous asset for all actors involved in the language classroom: students receive authentic, peer-related feedback on how to speak mathematics according to ideolect as well as register; tutors provide language teachers with meaningful and authentic mathematical texts, which can then serve as a basis for the development of course materials; teachers benefit from the content-based interaction between students and tutors and can focus their attention on linguistic issues and learning strategies relevant to all languages for special purposes and in particular the language of mathematics; through their interaction with students and with the language teacher, tutors can better understand that academic mathematical language must be learned, and that students need time to practice these structures and own this language. Given the CLIL experience, it is also expected that these tutors will be more likely to make adjustments later in their own classrooms to meet the language needs of their own students.

In the past the ultimate goal for language learners was to achieve the proficiency level of native speakers. This perspective has happily changed both in the field of second-language /multilingual acquisition research and in the language-learning classroom, as it has become abundantly clear that even native speakers themselves do not necessarily have or even need the ‘highest’ level of proficiency in their L1. Furthermore, native-like proficiency does not just depend on factors such as “nativeness”, it also develops based on a clear and reflected understanding of the requirements of the specific communicative situation at hand.

International Bachelor’s students in their first semester demonstrate a high level of German language proficiency (language tests); however, this language proficiency is academically general in nature and does not necessarily include skills in specific language purposes. In the same sense, native speakers begin their studies with similar linguistic skills, having little or no prior content-specific language preparation, even in a subject like first semester mathematics. This is despite the fact that almost all students in higher education will need these particular con-
text and content-specific skills both during their studies and beyond. Based on the real and on-going need for effective content-based oral communication at a local, national and international level, the time for implementing explicit CLIL training for students is now. Although the examples in this paper have focused on the field of mathematics, making use of content and language tutors in CLIL language courses in other major subject areas, including chemistry, biology, or physics, could pave the way for more skilled and better-prepared international university students.
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Appendix 1: Mathematical symbols identified as important i.e. useful for first-semester bachelor’s students attending Mathematics I (lecture and tutorial)

<table>
<thead>
<tr>
<th>Symbols Frequently Used in Mathematics I</th>
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<tbody>
<tr>
<td>+</td>
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<td>\mathbb{C}</td>
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</tbody>
</table>
2: Mathematical sentences containing symbols frequently used in first semester mathematics courses (Mathematics I)

1. \( \{3, 1, 5, 4\} \cup \{2, 5, 6\} = \{1, 2, 3, 4, 5, 6\} \)
2. \( [1, 4] \cap (2, 9) = (2, 4] \)
3. \( M = \{x \in \mathbb{R} | x > 10\} \)
4. \( (A \cup B) \setminus C \neq A \cup (B \setminus C) \)
5. \( M_1 \cap M_2 = \emptyset \Rightarrow \forall x \in M_1 : x \notin M_2 \)
6. \( M_1 \subset M \land M_2 \subset M \Rightarrow M_1 \cap M_2 \subset M_1 \cup M_2 \subset M \)
7. \( \sqrt{8} < \sqrt{9} < \left(\frac{1}{4}\right)^{-3} \)
8. \( 2 \neq 3^2 - 2^3 \)
9. \( \frac{3b}{a+1} + \frac{a}{2} = \frac{a^2 + a + 6b}{2a+2} \)
10. \( 5(2x - 3) - 7 = 28 \Leftrightarrow x = 2 \)
11. \( a^4 = (-2)^{6+2} \Leftrightarrow a = 4 \lor a = -4 \)
12. \( \forall y \in \mathbb{R} : (2 + 5y)(2 - 5y) = 4 - 25y^2 \)
13. \( \forall x \in \mathbb{Q} \exists p \in \mathbb{Z} \exists q \in \mathbb{N} : x = \frac{p}{q} \)
14. \( \forall \varepsilon > 0 \exists n \in \mathbb{N} : \varepsilon n > 1 \)
15. \( \forall a \in \mathbb{R} : -1 < a < 1 \Rightarrow a^2 < 1 \)
16. \( n \in \mathbb{N} \lor n \geq 3 \Rightarrow \forall a, b, c \in \mathbb{N} : a^n + b^n \neq c^n \)
17. \( x \in A \Leftrightarrow x \in \mathbb{N} \land \sqrt{x} \in \mathbb{N} \)
18. \( a_n = \frac{1}{n} \Rightarrow \forall n \in \mathbb{N} : a_{n+1} = a_n \)
19. \( f : [0, 1] \rightarrow [0, 30], \ x \mapsto (x + 2)^2 + 5x^3 \)
20. \( \dot{f} : \mathbb{R} \rightarrow \mathbb{R}, \ \dot{f}(x) = \frac{1}{1+x^2} \)

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